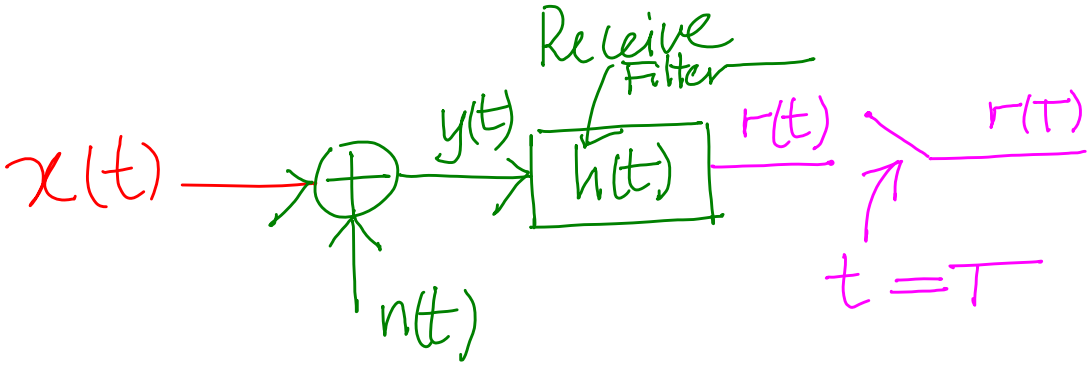


$$\text{Signal} = \int_{-\infty}^{\infty} a_0\, p(T-\tau)\, h(\tau)\, d\tau$$

$$\text{Noise} = \int_{-\infty}^{\infty} n(T-\tau)\, h(\tau)\, d\tau.$$

$$\text{SNR} = \frac{\text{Signal Power}}{\text{Noise Power}}$$

SNR: Signal to Noise Power ratio.

$$= \frac{P_d \left(\int_{-\infty}^{\infty} p(T-\tau)\, h(\tau)\, d\tau \right)^2}{\frac{\eta_0}{2} \int_{-\infty}^{\infty} |h(\tau)|^2\, d\tau}$$

We have to find filter $h(t)$ which maximizes SNR.

$$\left(\int_{-\infty}^{\infty} u(t)\cdot v(t)\,dt\right)^2 \le \int_{-\infty}^{\infty} u^2(t)\,dt \times \int_{-\infty}^{\infty} v^2(t)\,dt$$

Equality holds when $u(t) \propto v(t)$

$\Rightarrow u(t) = K v(t)$ (K: constant)

Cauchy – Schwarz Inequality

Using the Cauchy – Schwarz inequality, one can write,

$$SNR \le \frac{P_d \cdot \int_{-\infty}^{\infty} p^2(T-\tau)\,d\tau \cdot \int_{-\infty}^{\infty} h^2(\tau)\,d\tau}{\frac{\eta_0}{2}\int_{-\infty}^{\infty} h^2(\tau)\,d\tau}$$

$$\Rightarrow SNR \le \frac{P_d \cdot \int_{-\infty}^{\infty} p^2(T-\tau)\,d\tau}{\frac{\eta_0}{2}}$$

SNR is maximized when $p(T-\tau) \propto h(\tau)$

$\Rightarrow p(T-\tau) = Kh(\tau)$

Without loss of generality $K=1$

$$\Rightarrow \boxed{h(\tau) = p(T-\tau)}$$

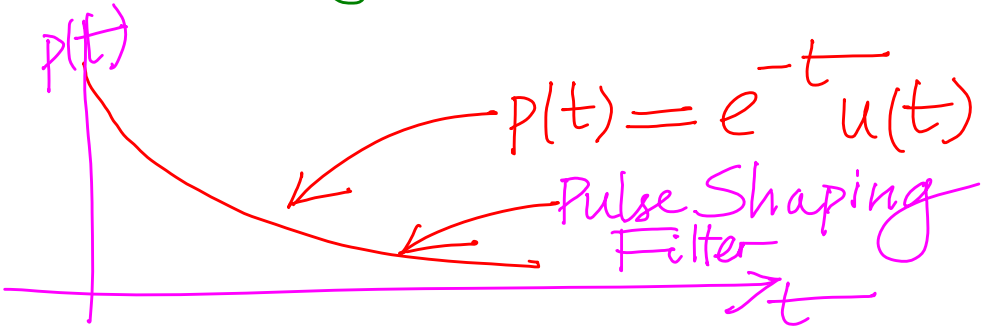
For maximum SNR

impulse response has to be matched to pulse Shaping filter. Hence termed as

Matched Filter

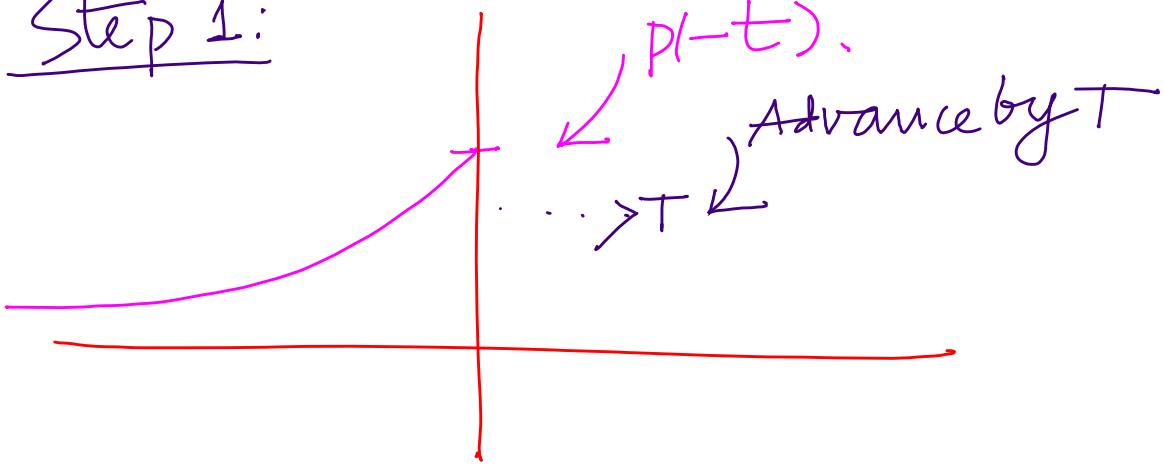
Matched Filter Principle,

To maximize SNR at receiver one has to employ matched Filter.



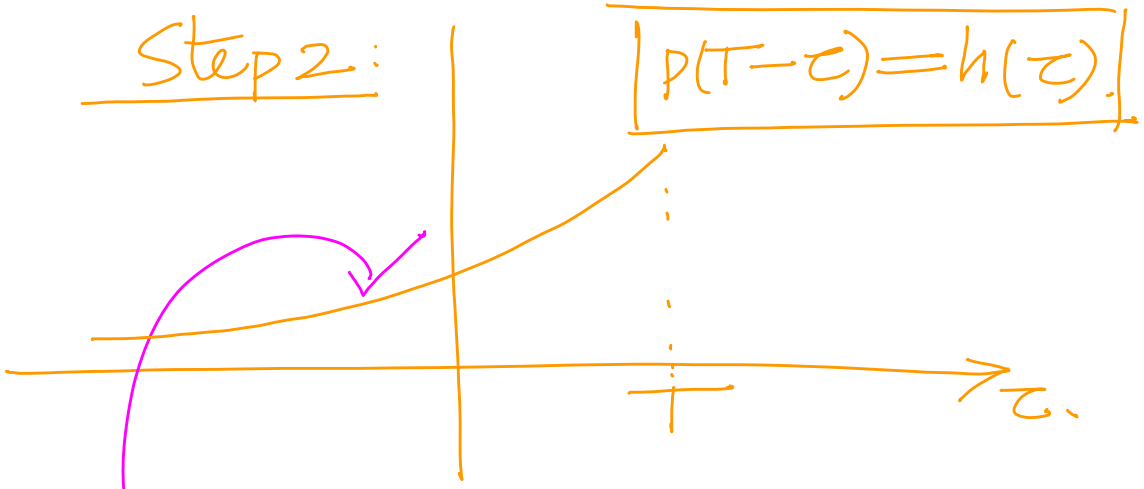
$$h(\tau) = p(T-\tau).$$

Step 1:



Step 2:

$p(T-\tau) = h(\tau)$.



Optimal Filter = Matched Filter which maximizes SNR.

Max SNR for Matched Filter

$$SNR = \frac{P_d \int_{-\infty}^{\infty} p^2(T-\tau)\,d\tau}{\frac{\eta_e}{2}}$$

$$= \frac{P_d \cdot \int_{-\infty}^{\infty} h^2(\tau)\,d\tau}{\frac{\eta_e}{2}}.$$

$$\int_{-\infty}^{\infty} p^2(T-\tau)\,d\tau$$

$$T-\tau = \tilde{\tau}$$

$$-d\tau = d\tilde{\tau}$$

$$\int_{\infty}^{-\infty} p^2(\tilde{\tau})(-d\tilde{\tau}).$$

$$= \int_{-\infty}^{\infty} p^2(\tilde{\tau})\,d\tilde{\tau}$$

$$= \text{Energy of Pulse Shaping Filter}$$

$$SNR = \frac{P_{d.}}{\left(\frac{\eta_o}{2}\right)} \int_{-\infty}^{\infty} h^2(\tau)\, d\tau$$

$$= \frac{P_{d.}}{\frac{\eta_o}{2}} \int_{-\infty}^{\infty} p^2(\tau)\, d\tau$$

Maximum SNR For matched Filter

$$\boxed{h(\tau) = p(T-\tau)}$$